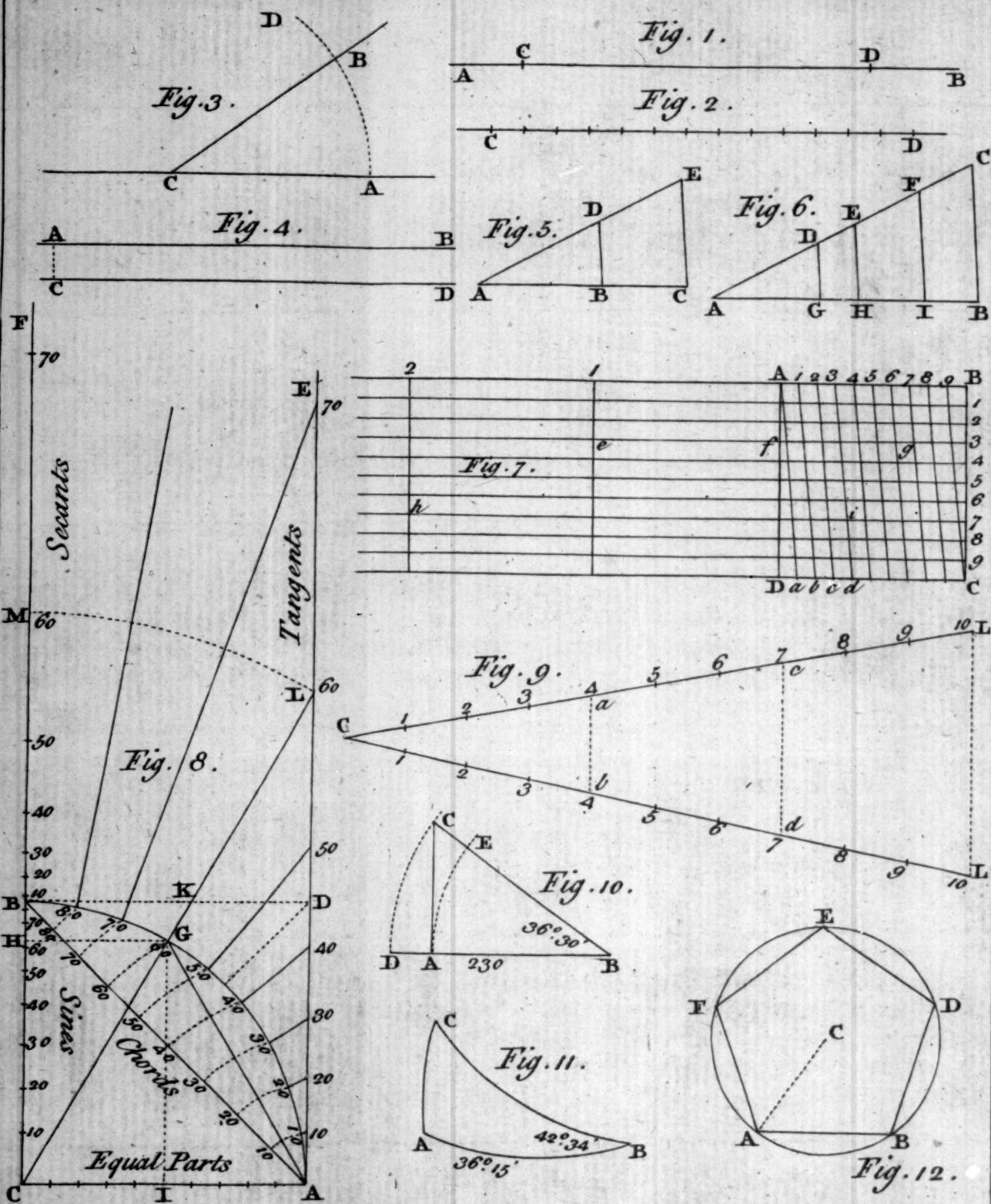
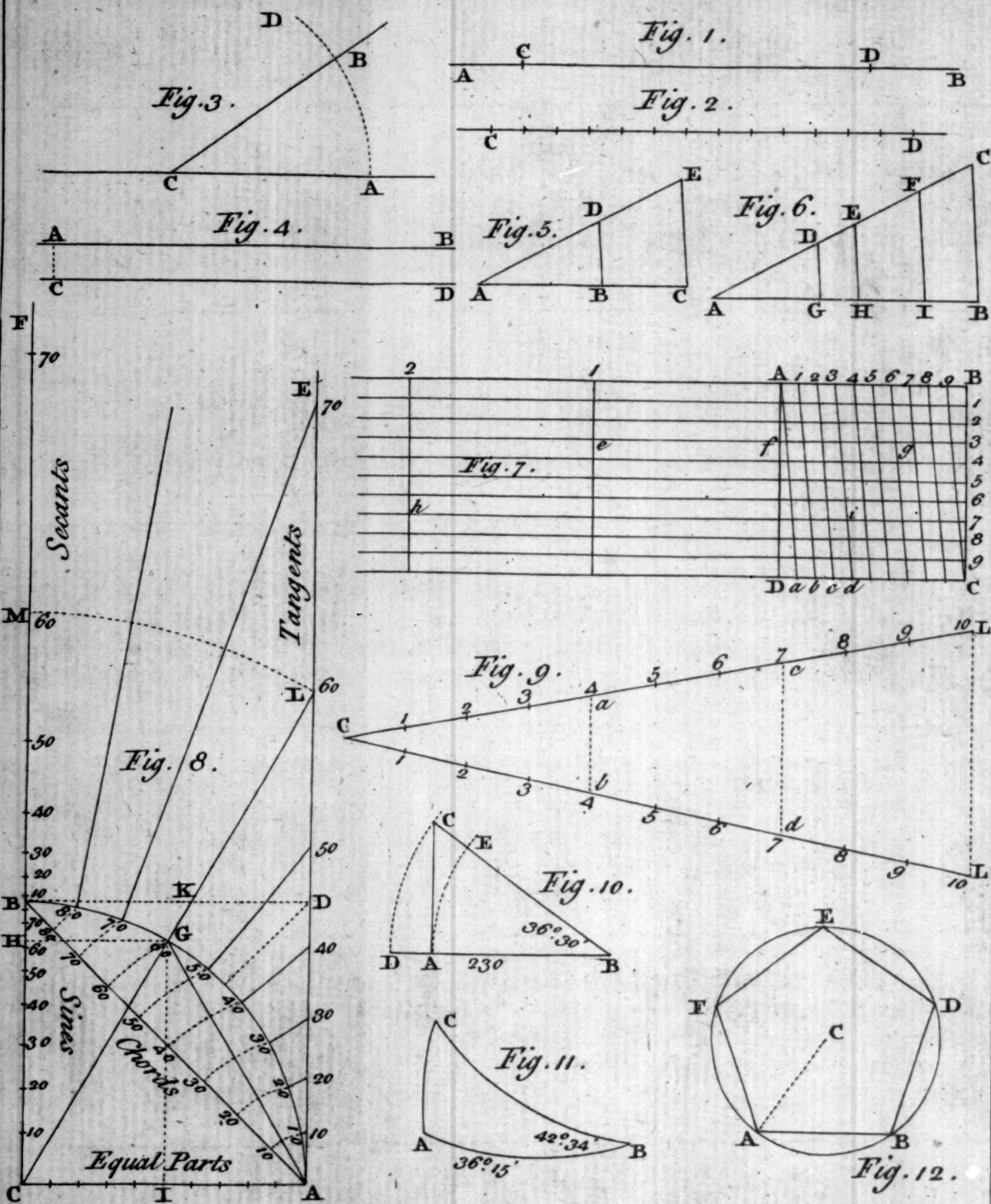


For a CASE of Mathematical INSTRUMENTS



For a CASE of Mathematical INSTRUMENTS



THE
DESCRIPTION and USE
OF
A CASE of MATHEMATICAL
INSTRUMENTS;

PARTICULARLY

Of all the LINES contained on the PLAIN
SCALE, the SECTOR, the GUNTER, and
the *Proportional* COMPASSES.

WITH

A Practical APPLICATION exemplified in many
useful Cases of GEOMETRY, and *Plain* and
Spherical TRIGONOMETRY.

The Whole illustrated by Copper-Plate Figures

BY BENJAMIN MARTIN.

L O N D O N:

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P R E F A C E.

I SHALL make no other Apology for troubling the Public with this small Tract, but only this, that it is a Small One; for the Treatises on this Subject, which I have hitherto seen, are, I think, too large for the generality of those who purchase Cases of Mathematical Instruments. And by what I am able to judge from many Years Experience, I am apt to believe, one more concise will be more acceptable and serviceable. For when the INSTRUMENTS are described, the Nature and Construction of the several LINES and SCALES fully declared, and the Use and Application of them to such practical Problems of GEOMETRY, and Plain and Spherical TRIGONOMETRY in their several Varieties, what Need can there be of any thing more? And so much you will find here. Indeed I have said nothing of the LINES of LATITUDES and HOURS, since, as they are solely appropriated to the Construction of DIALS, the Reader may find them explained and applied at large in my New ART of DIALLING. As to the SCALES used in NAVIGATION, they are new modelled, and explained in the MARINER'S MIRROR. And for a more full Illustration of the Common Sliding GUNTER, and a New Construction of the LINES of SINES and Tangents, see the DESCRIPTION and USE of a New SLIDING RULE with Double Slides, contiguous to each other. And, lastly, for a Description of a New Universal PROTRACTOR, for estimating Angles to a Minute; a new constructed PARALLEL RULER, applied to a Drawing TABLE; Three different Sorts of PANTAGRAPHs; with the SPIRIT-LEVEL, and its Use, I refer the Reader to my TREATISE on the GONIOMETER, or New ART of SURVEYING.

To conclude, I presume no one can now complain that he has it not in his Power to become acquainted with the Uses of all the considerable Mathematical Instruments, in the shortest Time, and with the least Expence, he can reasonably expect or desire.

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DESCRIPTION and Use of a CASE
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MATHEMATICAL INSTRUMENTS.

A POCKET CASE of *Mathematical* INSTRUMENTS contains the following Particulars, viz. (1.) A PAIR of *Plain* COMPASSES. (2.) A PAIR of *Drawing* COMPASSES, with its several Parts. (3.) A *Drawing* PEN and *Pointer*. (4.) A PROTRACTOR, in Form of a *Semicircle*, or, sometimes, of a *Parallelogram*. (5.) A PARALLEL RULER. (6.) A *Plain* SCALE. (7.) A SECTOR: Besides the *Black-Lead Pencil* for drawing Lines. The general Uses of the above Instruments are as follow.

I. Of the PLAIN COMPASSES. Fig. 1.

THE Use of the Common or *Plain* Compasses is (1.) To draw a *Blank Line* A B, by the Edge of a Ruler, through any given Point or Points C, D, &c. (2.) To any Extent or Length between the Points of the Compasses, and to set it off, or apply it successively upon any Line, as from C to D. (Fig. 2.) (3.) To take any proposed Line C D between the Points, and by applying it to the proper Scale, to find its
B Length

Length. (4.) To set off *Equal Distances* upon a given Line, by making a Dot with the Point at each, through which to draw parallel Lines. (5.) To draw any blank Circle, intersecting *Arches*, &c. (6.) To lay off an Angle of a given Quantity upon an Arch of a Circle from the Line of Chords, &c. (7.) To measure any Arch, or Angle, upon the Chords, &c. (8.) To construct any proposed Figure, in Plotting or Making Plans, &c. by setting off the Quantity of the Sides and Angles from proper Scales. In short, the Use of the Compasses occurs in every Branch of practical Mathematics, as we shall see more particularly hereafter.

II. Of the DRAWING COMPASSES.

These Compasses are chiefly designed for drawing Circles, and circular Arches; and it is often necessary they should be drawn with different Materials; and therefore this Pair of Compasses has, in one of its Legs, a triangular *Socket*, and *Screw*, to receive and fasten the following Parts or Points for that Purpose, viz. (1.) A *Steel Point*, which being fix'd in the Socket, makes the Compasses then but a *Plain Pair*, and has all the same Uses as just now described in drawing Blank Circles, setting off Lines, &c. (2.) A *Port Crayon* with a *black lead Pencil*, cut to a fine Point, for drawing Lines that may be easily rubb'd out again, if not right. A Piece of *Slate-Pencil* may be also used in this Part for drawing on *Slate*. (3.) The *Dotting Point*, or *Dotting Pen* with a small *Rowel*, or indented Wheel at the end, moving very freely; and receiving Ink from the brass Pen over it, communicates the same in equal and regular *Dots* upon the Paper, where dotted Lines are chosen. (4.) The *Steel Pen* or *Point*, for drawing and describing *Black Lines* with *Ink*; for this Purpose the two Parts or Sides of the Pen, are opened or closed with an adjusting Screw, that the Line drawn may be as fine or as coarse, as you please.

In

In the *Port-Crayon*, *Dotter*, and *Steel-Pen* there is a Joint, by which you can set the lower Part always *perpendicular* to the Paper, which is necessary for drawing a Line well, in every Extent or opening of the Compasses.

In some of the better Sort of Instruments, these Points slide into the Socket, and are kept tight by a Spring on the Inside that is not seen.

The *Steel Point* is sometimes made with a Joint, and furnished with a *fine Spring and Screw*; by which, when you have opened the Compasses nearly to the Extent required, you can by turning the Screw, move the Point to the true Extent, as it were, to a *Hair's Breadth*; which is the Reason these are called *Hair Compasses*.

The common Compasses, at large, are not altogether so well adapted for *small Drawings*; and therefore a small Sort, called *Bowes*, are contrived to answer all such Purposes; they consist only of a *Steel Point* and *drawing-Pen*, with a Joint, and of a small Length, so that very small Circles may be nicely drawn with them, as they are to be conveniently moved and turned about in the Hand, by a short Stem or Shaft.

III. Of the DRAWING PEN, and PENCIL.

The *Drawing Pen* is only the common Steel Pen at the End of a Brass Rod, or Shaft, of a convenient Length, to be held in the Hand for drawing all Kinds of *strait* Black Lines by the Edge of a Rule. The Shaft or Handle has a Screw in the middle Part; and when unscrewed, there is a *fine round Steel Pin, or Point*, by which you make as nice a Mark or Dot upon the Paper as you please, for terminating your Lines in curious Draughts.

The *Black-Lead Pencil*, if good, is of frequent Use for drawing *strait* Lines; and for supplying the Place of the

Drawing Pen, where Lines of Ink are not necessary; it is also often substituted for the *Common Pen* in *Writing*, *Figuring*, &c. Because in all Cases, if what be drawn with it, be not right, or does not please, it may be very easy rubb'd out with a Piece of Crum-Bread, and the whole new drawn.

IV. Of the PROTRACTOR.

The *Protractor* is a Semicircle of Brass, divided into 180 Degrees, and numbered each Way from End to End of the Semicircle by 10°, 20°, 30°, &c. The *Central Line* is the external Edge of the Protractor's Diameter, or strait Side; sloped down to the under Side, and is generally call'd a *fiducial Edge*; in the middle of which is a small Line, or fine Notch in the very Edge, for the *Center* of the Protractor. The Uses of the Protractor, are Two, (1.) To measure any Angle proposed. (2.) To lay down any Angle required.

For Example, Suppose it required to find what Number of Degrees are contained in the Angle ACB (Fig. 3.) You place the Center of the Protractor upon the Angular Point C, and the fiducial Edge exactly upon the Line CA; then observe what Number of Degrees the Line CB cuts upon the graduated Limb of the Protractor, and that will be the Measure of the Angle ACB, as required.

Secondly, suppose it required to protract or lay off from the Line AC, an Angle ACB equal 35 Degrees. To do this, You place the Center of the Protractor upon the given Point C, and the strait Edge upon AC very exactly; then make a fine Point or Dot at 35 Degrees on the Limb at B; and, the Protractor being removed, you draw through B, the strait Line CB, and it will make the Angle ACB, required.

Pro-

Protractors in Form of a *Parallelogram*, or Long Square, are usually made in Ivory or Brass, are more Exact than the common Semicircular Ones, for Angles to 40 or 50 Degrees; because at and about each End, the Divisions, (being farther from the Center) are larger; but the Advantage scarcely compensates the Expence.

V. Of the PARELLEL RULER.

The *Parallel Ruler* is so called, because as it consists of two strait Rules, connected together by two brass Bars, yet so as to admit a very free Motion to each; the One Ruler must always move *parallel-wise* to the other, that is, one Rule will be every where equidistant from the other, and by this Means it becomes naturally fitted for drawing one or more Lines *parallel to*, or equally distant from any Line proposed. The Manner of doing which is thus.

Let it be required to draw a strait Line, parallel to a given Line AB, and at the Distance AC from it. (Fig. 4.) First open the Rulers to a greater distance than AC, and place the Edge of one of the Rules exactly on the Line AB, then holding the other Rule (or Side) firmly on the Paper, you move the upper Rule down from A to the Point C, by which (holding it fast) you draw the Line CD, which will be parallel to the given Line AB, as required.

Many very useful Problems in the Mathematics are performed by this Instrument; of which the following are Examples.

Let it be required to find a *fourth Proportional* to three Right Lines given, AB, BC, and AD. (Fig. 5.) To do this, draw the Lines AC, AE, making an Angle at pleasure. Upon AC with the Compasses set off the Lines AB, and BC; and upon AE set off the Line AD; join DB,

DB, and parallel to it draw EC, then will DE be the 4th Proportional required. For $AB : BC :: AD : DE$.

Again, suppose it required to divide any Line AB as another Line AC is divided. (Fig. 6.) To do this, join the Extremities of each Line CB; and parallel to CB draw EI, EH, DG through the given Points D, E, F, in the Line AC; and by these Lines the Line AB will be divided exactly similar to the Line AC.

The Parallel Ruler is seldom put into a Case of Instruments, but those of the larger and better Sort; being generally sold by itself of various sizes, from 6 Inches to 2 Feet, in Length.

VI. *Of the PLAIN SCALE.*

The *Plain Scale* in common CASES of Instruments has the following Lines or Scales upon it, viz. (1.) A *Line of Six Inches*. (2.) A *Line of 50 equal Parts*. (3.) A *Diagonal Scale*. These are put on one Side. On the other Side are (4.) A *Line of Chords*, marked C. (5.) Seven particular Scales of Equal Parts, or *Decimal Scales*, of different Sizes; the Numbers placed at the beginning of each denote, how many of the small Divisions at the Beginning, are contained in one Inch, viz. 10, 15, 20, 25, 30, 35, 40.

The Use of the *Line of Inches* is the same in this as in all other Rules, viz. to take the Length or Dimensions of Bodies in *Inches*, and *Tenths* of an Inch, in order to compute their Contents.

The Line of 50 Equal Parts being equal to Six Inches, shews the *Foot* to be divided into 100 of the same Equal Parts, and the Divisions of this Line are placed by those of the Inches, that it may be easily seen what Number in One is equal to a given Number of the other; thus 3 Inches is equal to 25 Parts of the 100. And 30 of these latter are equal to 3 Inches and 6 Tenths. This Line is therefore often useful in practical Mathematics.

The



The *Diagonal Scale*, is properly a *Centessimal Scale*, because by it you divide an Unit into 100 equal Parts; and therefore you can lay off, or express, any Number to the 100th Part of an Unit, which is an Exactness generally sufficient in practical Business. How this is done will be easy to understand as follows. Let AB be 1 or Unit, and divide it into 10 equal Parts at 1, 2, 3, 4, &c. At a proper Distance BC, draw the Line CD equal and parallel to AB, and divide this Line CD also into 10 equal Parts at a, b, c, d, &c. then join the Points A a, 1 b, 2 c, 3 d, &c. and these will be the 10 *Diagonal Lines*. Lastly, divide BC into 10 equal Parts also, and number them 1, 2, 3, 4, &c. to 10 at C; then through each of these Divisions draw Lines parallel to AB, through the Length of the Scale, and the Construction is completed. See Fig. 7.

In this Diagonal Scale (upon the Plate) AB is One Inch; then if it be required to take off $1\frac{73}{100}$ Inches, or 1,73; set one Foot of the Compasses in the 3d Parallel under 1 at e, and extend the other Foot or Point to the 7th Diagonal in that Parallel at g; and the Distance eg is that required; for ef is 1 Inch, and fg is 73 Parts of 100.

Again, suppose it required to set off upon any Line 2,37 Inches; then place one Point of the Compasses on the 7th Parallel under 2 at h, and extend the other to the 3d Diagonal in the same Parallel at i; and the Distance hi is that required. Or if AB be 10, the Distance eg is 17,3; and hi is 23,7. Also, if AB be 100, then eg is 173, hi is 237; and so on.

This Diagonal Scale has this Centessimal Division at each End, and the Unit in one is just double of that at the other; thus if AB be 1 Inch at one End, it is $\frac{1}{2}$ an Inch at the other; Or if it be $\frac{1}{2}$ an Inch in the larger, it is $\frac{1}{4}$ Inch in the lesser Divisions, as is the Case upon most of the Common Plain Scales.

This

This Unit AB may also be 1 Foot, 1 Yard, 1 Rod, 1 Mile, &c. So that every Unit in every Kind of Measures is hereby estimated in 1000th Parts of the Whole, which shews the *Diagonal Scale* to be a most useful Invention.

On the other Side of the Plain Scale, are the Seven Decimal Lines, which are usually called *Plotting Scales*, because their Divisions of an Unit into 10 Parts being different in Proportion of 4 to 1, the Surveyor has it in his Power to vary the Scale of his Plot, or Plan of an Estate, &c. in that Ratio, in Seven different Drawings; and the Superficies or Sizes of the greatest and least Plans will be as 16 to 1. Or that drawn by a Scale N°. 10, will be 16 Times larger than the Plan laid down from Scale N°. 40.

Also the same Variety is to be had in the Construction of all other Geometrical Figures, whether *Superfices*, or *Solids*; and with Respect to the latter, the greatest will be to the least as 64 to 1; That is, the *Architect* can vary the Size of his *House*, *Temple*, &c. in the Ratio of 64 to 1 in 7 different Elevations.

The last *Line* on the common Plain Scale, is that of *Chords*; and is much more used than the Protractor for laying off, or measuring any proposed Angle. Thus let it be required to draw the Line BC to make an Angle of 35 Degrees with the Line AC (Fig. 3.) to do this, set one Point of the Compasses in the Beginning of the Line of Chords, and extend the other to 60; with that Extent (as Radius) place one Foot in C, and with the other describe the Arch AD; then take from the Chords 35°. in the Compasses, and set them on the Arch from A to B; and through B, draw CB, and it is done.

Again, suppose it required to measure any Angle ABC proposed, proceed thus; produce CA indefinitely, take
60°.

60° from the Chords in the Compasses, and with one Foot in C strike the Arch AD, cutting the Leg BC in B; then take the Arch AB in the Compasses, and applying it upon the Beginning of the Line of Chords, it will reach to 35°. the Quantity of the Angle required. But the *Line of Chords* is more useful on the *Sector*, to which we now proceed.

Note. This Line of Chords is so constructed on my *Navigation Scales*, with Sexagesimal Divisions, that any Angle may be measured, or laid off, to a Minute of a Degree.

VII. Of the SECTOR:

The *Sector* is the most useful of all Mathematical Instruments; for it not only contains all the most useful Lines or Scales, but, by its Nature, renders them universal, as we shall shew in the following Description.

The Lines commonly laid down upon the Sector are these (1.) A *Line of Equal Parts*, marked L at the End. (2.) A *Line of Chords* to 60 Degrees, marked C. (3.) A *Line of Sines*, 90 Degrees, marked S. (4.) A *Line of Tangents*, to 45°. marked T. (5.) Another Line of Tangents from 45 to 75 or upwards, marked *ta*. (6.) A *Line of Secants*, marked *se*. (7.) A *Line of Polygons*, marked POL.

Besides these, when the Sector is quite opened, there is on one Side, (1.) *Gunter Line of Artificial Numbers*, *n*. (2.) *Line of Artificial Sines*, *s*. (3.) *Line of Artificial Tangents*, *t*. There is also a Line of 12 Inches, and another of the Foot divided into 1000 Equal Parts, placed by it, for the Purposes already mentioned.

Before a proper Idea can be formed of these Sectoral Lines, and their Uses, we must shew their Construction

C

from

from the Circle. To this End, let AGB , (Fig. 8.) be a Quarter of a Circle divided into 90 Degrees, described with the Radius AC , on the Center C ; let AF and CF be perpendicular to AC , in A and C ; then if the Radius AC be divided into 10, 100, 1000, &c. *Equal Parts*, it will be the Line so called upon the Sector.

If from A , a Line be drawn to any Part or Division of the Quadrant as G at 60° , then that Line AG is the Chord of that Arch, or of 60° . And if the Line AB be drawn, it will be the Chord of 90° ; and by setting one Foot of the Compasses in A , and extending the other to the several Divisions 10, 20, 30, 40, &c. they may be transferred from the Circle to the Line AB , which will then be properly divided into a *Line of Chords* in 10, 20, 30, 40, &c. to 90, as on the Plane Scale.

If from any Point G in the Quadrant you let fall a Perpendicular GI to the Radius AC , or GH to the Radius CB , then the Line GI is called the *Sine* of the Arch AG ; and the Line GH is the *Sine* of the Arch GB , the Complement of AG to 90° . And if all the Divisions of the Quadrant were transferred to CB by Lines GH parallel to AB ; then the Line CB will be divided as a *Line of Sines* in the Points 10, 20, 30, 40, &c. to 90° , as on the Sector you see it.

By laying a Rule from the Center C to the several Divisions of the Quadrant 10, 20, 30, &c. it will cut the Line AE in the Points 10, 20, 30, &c. which will be thereby divided into a *Line of Tangents*; and here it must be observed that the Line of Tangents T on the Sector, extends but to 45° equal to AD (or BC) Radius. And that the Line of lesser Tangents t , are projected from a lesser Radius, and begin from 45° at the Distance of its Radius from the Center of the Sector.

By

By drawing the Line CL through the Division 60° , to the Line AE, it makes AL the *Tangent* of 60° , and is itself the *Secant* of 60° , or of the Angle ACL. And if with one Foot of the Compasses in C you extend the other to the several Divisions in the Line AE, and transfer them to the Line CE; then will the Part BF be thus divided into a *Line of Secants*; being placed at the Distance of the Radius CB, from the Center of the Sector, and beginning at B, where the Radius ends.

It may be of Use in many Cases to observe, (1.) That the *Chord* of 60° AG, is equal to Radius AC or CG. (2.) That the *Sine* of 60° GI, bisects the Radius AC in I; and therefore the *Sine* GH of 30° is equal to $\frac{1}{2}$ the Radius or CI. (3.) And therefore the *Secant* CL of 60° , is equal to twice the Radius AC; for CI is to CG as CA to CL. (4.) Therefore *Cofine* is to *Radius* as *Radius* to the *Secant*. (5.) Also the *Tangent* AL is to Radius AC as Radius BC is to the *Co-tangent* BK.

From what has been said, the Reason appears why the *Line of Lines* (or equal Parts L, terminates upon the Sector at 10; the *Line of Chords* C at 60° ; the *Line of Sines* S at 90° ; the larger *Tangents* T at 45° ; and that the *lesser Tangents*, and also the *Secants*, are of indefinite Length.

From the Nature of the Sector, consisting of two *Pairs*, or *Legs*, moveable upon a central joint, it is requisite that the Lines should be laid on the Sector by Pairs, viz. one of a Sort on each Leg, and all of them issuing from the Center, all of the same Length, and every Two containing the same Angle. We shall now illustrate the Nature of working Problems *Sectoral-wise* as follows, by the *Lines of Lines* or *Equal Parts* LL.

Let CL, CL. (Fig. 9.) be the two *Lines of Lines* upon the Sector, open'd to an Angle LCL; join the

Divisions 4 and 4, 7 and 7, 10 and 10, by the dotted Lines ab , cd , LL . Then by the Nature of similar Triangles, it is CL to Cb , as LL to ab ; and CL to Cd , as LL to cd ; and therefore ab is the same Part of LL as Cb is of CL . Consequently, if LL be 10, then ab will be 4, and cd will be 7 of the same Parts.

And hence though the *Lateral Scale* CL be fixed, yet a *Parallel Scale* LL , is obtainable at Pleasure; and therefore though the *lateral Radius* is of a determined Length in the Lines of *Chords*, *Sines*, *Tangents* and *Secants*, yet the *Parallel Radius* may be had of any Size you want, by means of the Sector, as far as its Length will admit; and all the *Parallel Sines*, &c. peculiar to it; as will be evident by the following Examples in each Pair of Lines.

EXAMPLE I. In the LINES OF EQUAL PARTS.

Having 3 Numbers given 4, 7, 16, to find a 4th Proportional. To do this, take the *lateral Extent* of 16 in the Line CL , and apply it *Parallel-wise*, from 4 to 4, by a proper opening of the Sector; then take the parallel Distance from 7 to 7 in your Compasses, and applying one Foot in C , the other will fall on 28 in the Line of Lines CL , and is the Number required; for $4 : 7 :: 16 : 28$.

EXAMPLE II. In the LINES OF CHORDS.

Suppose it required to lay off an Angle ACB , (Fig. 3.) equal to 35° ; then with any convenient opening of the Sector, take the Extent from 60 to 60, and with it (as Radius) on the Point C describe the Arch AD indefinitely; then in the same opening of the Sector) take the parallel Distance from 35° to 35° , and set it from A to B in the Arch AD , and draw AB and it makes the Angle at C required.

EXAMPLE

EXAMPLE III. In the LINES OF SINES.

The Lines of *Sines*, *Tangents*, and *Secants*, are used in Conjunction with the *Lines of Lines* in the Solution of all the Cases of Plain Trigonometry; thus, let there be given in the Triangle ABC, (Fig. 10.) the Side AB = 230; and the Angle ABC = $36^{\circ} 30'$; to find the Side AC. Here the Angle at C is $53^{\circ} 30'$. Then take the lateral Distance 230, from the *Line of Lines*, and make it a Parallel from $53^{\circ} 30'$ to $53^{\circ} 30'$ in the *Line of Sines*; then the parallel Distance between $36^{\circ} 30'$ in the same Lines, will reach laterally from the Center to 170,19 in the *Line of Lines* for the Side AC required.

EXAMPLE IV. In the LINES OF TANGENTS.

If instead of making the Side BC Radius (as before) you make AB Radius; then AC, which before was a *Sine*, is now the Tangent of the Angle B; and therefore to find it, you use the *Lines of Tangents*, thus.

Take the lateral Distance 230 from the *Line of Lines*, and make it a parallel Distance on the Tangent Radius, viz. from 45° to 45° . then the parallel Tangent from $36^{\circ} 30'$ to $36^{\circ} 30'$, will measure laterally on the *Line of Lines* 170,19, as before, for the Side AC.

EXAMPLE V. In the LINES OF SECANTS.

In the same Triangle, the Base AB, and the Angles at B and C given, as before, to find the Side or Hypotenuse BC. Here BC is the Secant of the Angle B.

Take the lateral Distance 230 on the *Line of Lines*, make it a parallel Distance at the Radius or Beginning of the *Lines of Secants*; then the parallel Secant of $60^{\circ} 30'$ will measure laterally on the *Line of Lines* 287,12, for the length of BC, as required.

EXAMPLE

EXAMPLE VI. In the LINES OF SINES AND TANGENTS conjointly.

In the Solution of Spherical Triangles, you use the *Lines of Sines* and *Tangents* only, as in the following Example. In the Spherical Triangle ABC (Fig. 11.) right angled at A, there is given the Side $AB = 36^{\circ} 15'$, and the adjacent Angle $B = 42^{\circ} 34'$, to find the Side AC. The Analogy is Radius : Sine of AB :: Tangent of B : Tangent of AC ; therefore make the lateral Sine of $36^{\circ} 15'$ a Parallel at Radius, or between 90 and 90 ; then the Parallel Tangent of $42^{\circ} 34'$ will give the lateral Tangent of $28^{\circ} 30'$ for the Side AC.

EXAMPLE VII. In the LINES OF POLYGONS.

It has been observed that the Chord of 60 Degrees is equal to Radius; and 60° is the 6th Part of 360° ; therefore such a Chord is the Side of a *Hexagon*, inscribed in a Circle. So that in the *Line of Polygons*, if you make the Parallel Distance between 6 and 6, the Radius of a Circle, as AC (Fig. 12.) then if you take the Parallel Distance between 5 and 5, and place it from A to B, the Line AB will be the Side of a *Pentagon* ABDEF, inscribed in the Circle; in the same Manner may any other Polygon from 4 to 12 Sides, be inscribed in a Circle, or upon any given Line AB.

VIII. OF GUNTER'S LINES.

We have now shewn the Use of all that are properly call'd *Sectoral Lines*, or that are to be used *Sector-wise*: But there is another Set of Lines usually put upon the Sector, that will in a more ready and simple Manner give the Answers to the Questions in the above Examples, and these are called *Artificial LINES* of NUMBERS, SINES, and

and TANGENTS; because they are only the Logarithms of the natural Numbers, Sines and Tangents, laid upon Lines or Scales, which Method was first invented by Mr. Edmund GUNTER, and is the Reason why they have ever since been called *Gunter's Lines*, or the *Gunter*.

Logarithms are only the *Ratios of Numbers*, and the Ratios of all proportional Numbers are equal. Now all Questions in *Multiplication*, *Division*, the *Rule of Three*, and the *Analogies of Plain and Spherical Trigonometry*, are all stated in proportional Numbers or Terms; therefore, if in the Compasses you take the Extent (or Ratio) between the first and second Terms, that will always be equal to the Extent (or Ratio) between the 3d and 4th Terms; and consequently, *if with the Extent between the 1st and 2d Terms, you place one Foot of the Compasses on the Third Term, then turning the Compasses about, the other Foot will fall on the Fourth Term sought.*

Thus in Example I. of the three given Numbers 4, 7, and 16, if you take the Extent from 4 to 7 in the Compasses, and place one Foot in 16, the other will fall on 28 the Answer, in the Line of Numbers mark'd *n*.

Again, the *Artificial Line of Numbers and Sines* are used together in Plain Trigonometry, as in Example III. where the two Angles B and C, and the Side AB are given, for here if you take the Extent of the two Angles $53^{\circ} 30'$ and $36^{\circ} 30'$ in the Line of Sines, mark'd *s*, then placing one Foot upon 230 in the Line of Numbers *n*, the other will reach to 170,19 the Answer.

Also the *Line of Numbers and Tangents* are used conjointly, as in Example IV. for take in the Line of Tangents *t*, the Extent from 45° (Radius) to $36^{\circ} 30'$, that will reach from 230 to 170,19 the Answer as before.

Lastly, the *Artificial Lines of Sines and Tangents* are used together in the Analogies of *Spherical Triangles*.

Thus

Thus Example VI. is solved by taking in the Line of Sines *s.* the Extent from 90° (Radius) to $36^\circ 15'$, then that in the Line of Tangents *t.* will reach from $42^\circ 34'$ to $28^\circ 30'$, the Answer required.

I shall only further observe that each Pair of Sectoral Lines contain the same Angle, viz. 6 Degrees in the common 6 Inch Sector; therefore to open these Lines to any given Angle, as 35° for Instance, you have only to take 35° laterally from the *Line of Chords*, and apply it *parallel-wise* from 60° to 60° in the same Lines, and they will all be opened to the given Angle of 35° .

If to the given Angle 35° you add the Angle 6° , which they contain, the Sum is 41° ; then take 41° laterally from the Line of Chords, and apply it Parallel from 60 to 60, then will the Sides or Edges of the Sector contain the same Angle of 35 Degrees. And in this Case the Sector becomes a general *Recipe-Angle*, which is an Instrument for taking the Quantity of any Angle contain'd between two *inclining Planes*, as those in *Fortifications*, &c.

IX. Of PROPORTIONAL COMPASSES.

Though this Sort of Compasses does not pertain to a Common Case of Instruments, yet a short Account of their Nature and Use may not be unacceptable to those who are not acquainted with them. They consist of two Parts or Sides of Brasses, which lie upon each other, so nicely as to appear but One, when they are shut. These Sides easily open, and move about a Center, which is itself moveable in a hollow Canal cut through the greatest Part of their Length. To this Center on each Side is affixed a Sliding Piece of a small Length, with a fine Line drawn on it serving as an *Index*, to be set against
other



other Lines or Divisions placed upon the Compasses on both Sides. These Lines are, (1.) A LINE of LINES. (2.) A LINE of SUPERFICES, AREAS, or PLANS. (3.) A LINE of SOLIDS. (4.) A LINE of CIRCLES, or rather of *Polygons* to be inscribed in Circles.

These Lines are all unequally divided, the Three first from 1 to 10; the last from 6 to 20; their Uses are as follows.

By the LINE of LINES you *divide a given Line into any Number of Equal Parts*; for by placing the Index against 1, and screwing it fast, if you open the Compasses, then the Distance between the Points at each End will be equal.—If you place the Index against 2, and open the Compasses, the Distance between the Points of the longer Legs will be twice the Distance between the shorter ones; and thus a Line is *bisected*, or divided into 2 equal Parts.—If the Index be placed against 3, and the Compasses opened, the Distances between the Points, will be as 3 to 1, and so a Line is divided into 3 *equal Parts*; and so you proceed for any other Number of Parts under 10.

The Numbers of the LINE of PLANS answer to the *Squares* of those in the Line of Lines; for because *Superficies or Plans are to each other as the Squares of their Like Sides*, therefore if the Index be placed against 2 in the Line of Plans; then the Distance between the small Points will be the Side of a Plan whose Area is 1; but the Distance of the larger Points will be the like Side of a Plan whose Area is 2, or twice as big.—If the Index be placed at 3, and the Compasses opened, the Distances between the Points at each End will be the *like Sides of Plans*, whose Areas are as 1 to 3, and so of others.

The Numbers of the LINE of SOLIDS answer to the *Cubes* of those in the *Line of Lines*; because *all Solids are*

to each other as the Cubes of their like Sides or Diameters; therefore, if the Index be placed to N^o 2, 3, 4, &c. in the *Line of Solids*, the Distances between the lesser and larger Points will be the like Sides of Solids which are to each other as 1 to 2, 1 to 3, 1 to 4, &c. For Example, if the Index be placed at 10, and the Compasses be open'd, so that the small Points may take the Diameter of a Bullet weighing 1 Ounce, then the Distance between the larger Points will be the Diameter of a Bullet or Globe of 10 Ounces, or which is 10 times as big.

Lastly, the Numbers in the *LINE OF CIRCLES* are the *Sides of Polygons* to be inscribed in a given Circle, or by which a Circle may be divided into those Equal Parts from 6 to 20. Thus if the Index be placed at 6, the Points of the Compasses at either End, when opened to the Radius of a given Circle, will contain the Side of a *Hexagon*, or divide the Circle into 6 equal Parts.—If the Index be placed against 7, and the Compasses opened, so that the larger Points may take in the Radius of the Circle; then the shorter Points will divide the Circle into 7 equal Parts for inscribing a *Heptagon*.—Again, placing the Index to 8, and opening the Compasses, the larger Points will contain the Radius, and the lesser Points divide the Circle into 8 equal Parts, for inscribing an *Octagon*, or *Square*. And thus you proceed for others.

A D V E R T I S E M E N T.

GENTLEMEN and LADIES may be supplied by the AUTHOR with all Kinds of *Philosophical, Mathematical, Optical, Nautical, &c. INSTRUMENTS*, at the most reasonable Prices; constructed as their THEORIES direct; and of a Form the most convenient for Practice; with Books describing their various Uses in the several ARTS and SCIENCES to which they are appropriated, written by the AUTHOR.

